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# ECONOMICS

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## BRAZIL

# What to Expect When You Are Expecting

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- **Brazil is approaching the moment when the inflation target for 2011 will be set, and there are already signs that the government intends to keep the target unchanged from the levels set in previous years. The apparent reason for that is the fear that a tighter inflation target for 2011 could force the Central Bank to hike rates in the second half of 2010, during political campaign season.**
- **We, however, believe that these fears are misplaced. They appear to stem from the notion that there is a stable trade-off between economic activity and inflation (hence a trade-off between inflation and real interest rates). However, developments in macroeconomic theory (as well as the empirical evidence) suggest that this stable trade-off does not exist. Indeed, the trade-off applies only when inflation expectations are fixed, but in real life inflation expectations are not a constant, but rather a variable that reacts, among other things, to the Central Bank stance in terms of monetary policy.**
- **Precisely for this reason, it is crucial to understand how inflation expectations would react to changes in the inflation target. In this report we show that current inflation expectations depend upon both the current target and future expected inflation (sometimes past inflation as well), and could be reduced if the government decides on a lower inflation target in the future. This feature prevails in a model in which inflationary persistence (inertia) is left aside, and even in a model that explicitly incorporates the inflationary inertia issue.**
- **To the extent that current inflation expectations react positively to the setting of a lower inflation target, the Central Bank finds additional room to reduce real interest rates, and output increases. These effects are, of course, temporary, but in a period during which the economy is struggling to recover from a deep negative shock to growth, lower inflation expectations and lower real interest rates can provide a welcome additional boost.**
- **Finally, given that the country, according to both market consensus and Central Bank forecasts, should record inflation rates below the target in 2009 and 2010, it can seize the opportunity to consolidate lower inflation rates below the levels recorded in the past few years.**


The decree that created and regulates the inflation targeting regime in Brazil requires that, in June of any given year, the National Monetary Council (CMN) should set the target for two years ahead – that is, in June 2009 the CMN must set the inflation target for 2011. Although a few weeks remain until the decision is expected to announced (it is scheduled for June 29), the debate is already under way. The government has already signaled that it will maintain the current target (4.5%, with a tolerance interval of 2 percentage points, from 2.5% to 6.5%), fearing that a lower target for 2011 could lead to tighter monetary policy in the second half of 2010[1] – close to the general election, which includes not only the House and the Senate, but also the choice of the new president and all the state governors (as well as state assemblies).

We, however, believe this argument does not really apply. Crucially, it relies on the assumption that there is a stable trade-off between inflation and growth (hence a trade-off between inflation and real interest rates), but the developments of monetary economics during the past 30 years suggest that this trade-off is far from stable. Indeed, macroeconomists who specialize in the monetary side of the economy have known for a while that the trade-off is valid only for a *given level of inflation expectations*.

[1] According to the newspaper *O Estado de São Paulo* (May 23, 2005), an anonymous government source stated that "economic activity should recover an accelerated pace in 2010 and a lower target could lead to *undesirable monetary tightening* [emphasis added], something that no one likes to observe, chiefly in an election year." The full quote (in Portuguese) can be found at http://clippingmp.planejamento.gov.br/cadastros/noticias/2009/5/23/meta-de-inflacao-devera-ser-mantida.

In other words, if inflation expectations remain unchanged, then to reduce inflation it is also necessary to push actual output below potential, which is achieved via higher interest rates.

Inflation expectations, however, need not remain unchanged, and in general they are anything but constant. Indeed, inflation expectations depend on the Central Bank approach to monetary policy, together with other potentially important variables that we explore in this note. If the Central Bank adopts policies that are consistent with inflation fluctuating around the target[2] it is possible to show that it can anchor inflation expectations at the target, either immediately, or taking a while longer in case inflation displays some persistence. Under such circumstances, special attention should be devoted to the issue of expectations.

It turns out, though, that the inflation target itself plays a central role in determining inflation expectations, not only for the year to which it is applicable, but also in earlier years. As we attempt to show, the setting of a future inflation target *below* the level at which it was expected to be set leads to a reduction of inflation expectations for both future periods and the present period. Nevertheless, as current inflation expectations drop, the Central Bank can find additional room for reducing real interest rates, leading to a (temporary) acceleration of growth. This means that, contrary to popular belief, a lower inflation target can actually have expansionary effects on output.

We show this first in a model that, for the sake of simplicity, leaves aside the issue of inflation persistence (or inertia). This model suggests that, provided the Central Bank follows a determined monetary rule, inflation expectations should be equal to the inflation target all the time. In this setting, the setting of a lower future inflation target implies a downward revision of future and current inflation expectations, so the latter allows for reduction in real interest rates and an increase in output.

Introducing inflationary inertia changes some of the results above, but not the fundamental point. In this case inflation expectations need not be equal to the target all the time, but can reflect the impact from past inflation as well. That said, even in this case it is possible to show that the decision to reduce the target for future inflation translates into a decline of current inflation expectations, and, as in the previous case, a better performance in terms of output, made possible by lower real interest rates. In both cases these effects materialize only in the current period; it is not possible to accelerate growth persistently by lowering the inflation target. Nevertheless, chiefly in a period during which the economy is struggling to recover from a negative shock, this extra boost would definitely be welcome. Moreover, in light of expectations of low inflation in 2009 and 2010 the country could seize the opportunity to consolidate inflation below the levels observed in previous years.

In all, both models suggest that fears of a possible additional tightening due to a lower target are misplaced. This view ignores the role of expectations and the possible positive effects of lower inflation targets.

Although we have attempted to confine the more complex material to the appendix, there is some algebra in the next two sections, which might require a more demanding reading than we generally prefer. Economists may like that, while non-economists may loathe it, but the truth is that we could not find any other way to keep track of all the interactions that can take place even in this simplified setting. In any case, we have put some effort into providing the economic intuition behind the equations, so that the reader should be able to understand the essential points of our story.

## The model without inflationary inertia

Our toy economy can be described by a set of three equations. The supply side of the economy is represented by a standard Phillips curve, according to which current inflation ($\pi_t$) depends on expectations about future inflation ($E_t\pi_{t+1}$), the output gap ($y_t$), and a supply shock ($e_t$), this last one assumed to have zero mean and a finite variance.:

$$\pi_t = E_t\pi_{t+1} + \alpha y_t + e_t \quad ; \alpha>0 \qquad (1)$$

Note that in the current case inflation depends only on the forward-looking component. The story behind an equation such as (1) refers to an economy in which companies adjust prices at given intervals (say, one year), and have to keep prices fixed during that interval. Thus, if a certain company is making its annual price adjustment, knowing that it will have to keep prices unchanged for the next 12 months, it should incorporate into the current price some expectation about how high inflation will be during that period. If inflation is expected to be high, the company will try to set prices higher, allowing for the inflationary erosion of its real price during the interval between adjustments. Conversely, in a low inflation environment, companies need not adjust prices so aggressively, as the erosion from inflation becomes a less serious issue.

The demand side of the economy is given by a simple IS relation: the output gap (that is, the difference between actual and potential output) is assumed to be negatively affected by the difference between the real interest rate, i.e., the nominal interest rate ($i_t$), deducted inflation expectations for the next 12 months ($E_t\pi_t$), and the neutral interest rate ( $\bar{r}$ ). As in the Phillips curve, there is a demand shock ($u_t$), with similar properties: zero mean and finite variance.

[2] Inflation should not typically be at the target all the time, since it is continually affected by demand and supply shocks that tend to push in various directions. Moreover, given that in Brazil the assessment of the relationship between actual inflation and the target takes place every December, the chances that inflation would be precisely at the target are quite small. But the point of the regime is not to make sure that every December inflation reaches the precise target, but rather to guarantee that deviations from the target are not persistent; inflation may reach below the target in a given year and above the target in another, remaining, however, *on average* at the target.

$$y_t = -\beta(i_t - E_t\pi_t - \bar{r}) + u_t \text{ ; } \beta>0 \qquad (2)$$

We finally assume that the Central Bank behavior can be described by a Taylor type rule, linking the choice of the nominal interest rate to the deviations of inflation expectations from the target ($\bar{\pi}$) and the output gap. In particular, the Central Bank increases the nominal rate above the nominal neutral rate (defined as the sum of the real neutral rate and inflation expectations for the next 12 months) whenever expected inflation is above the target, or when the output gap is positive (actual output reaches above potential).

$$i_t = \bar{r} + E_t\pi_t + a(E_t\pi_t - \bar{\pi}) + by_t \text{ ; } a>0;\ b>0 \qquad (3)$$

Notice that we assumed the parameter "a" in equation (3) to be positive, an assumption with important implications. Indeed, under this hypothesis the Central Bank reacts to changes in inflation expectations by more than 1:1 (that is, an increase of, say, 1% in inflation expectations implies an increase in interest rates of more than 1%). Acting this way the Central Bank not only compensates for the increase in inflation expectations on the real interest rate, but also pushes it up, leading to lower demand and activity, and hence to lower inflation. This pattern (known in the monetary economics literature as the "Taylor Principle") is a crucial condition for inflation stability, as we should see ahead.

Inserting the Taylor rule (3) into the IS equation (2) we can express the expected output gap in terms of the difference between inflation expectations and the inflation target:

$$E_t y_t = \frac{-\beta a}{1+b\beta}(E_t\pi_t - \bar{\pi}) \qquad (4)$$

Whenever inflation expectations are above the target the expected output gap is negative – i.e., actual output is below potential. The reason is straightforward: if the Central Bank expects inflation to be higher than the target, it increases the real interest rate (following the Taylor rule) above the neutral level, which, according to equation (2), translates into lower demand and a negative output gap. Conversely, if inflation expectations go below the target, the Central Bank cuts nominal rates, leading to a decline of the real interest rate and a positive output gap. This last consideration plays a central role in our argument, as we should make clear in this section and the next one.

Taking expectations of (1) and using equation (4) we can express current inflation expectations ($E_t\pi_t$) as a weighted average between future inflation expectations ($E_t\pi_{t+1}$) and the inflation target.

$$E_t\pi_t = \omega E_t\pi_{t+1} + (1-\omega)\bar{\pi} \qquad (5)$$

where $\omega = \dfrac{1+b\beta}{1+\beta(b+a\alpha)}$.

Notice that in the case a=0 (that is, when the Taylor Principle is not observed) the weight ($\omega$) on future inflation expectations becomes 1, while the weight on the inflation target collapses to zero. Under these circumstances inflation becomes undetermined (inflation tomorrow would be equal to inflation today plus a random shock), indicating that the observance to the Taylor Principle is a key condition for inflation stability, understood here as inflation hovering around the target, or, more precisely, inflation expectations equal to the inflation target.

Assuming a constant inflation target $\bar{\pi}$ from today until the crack of doom, it is possible to show that the solution to equation (5) implies:

$$E_t\pi_t = \bar{\pi} \qquad (6)$$

In plain words, by adhering to the Taylor Principle central banks can anchor inflation expectations at the target. This means that actual inflation should fluctuate around the target, affected, of course, by unpredictable demand and supply shocks, but not drifting away from the target over time. Note also that, as inflation is expected to be, on average, at the target, the output gap is expected, also on average, to be zero, according to equation (4), which, using (6), becomes:

$$E_t y_t = 0 \qquad (7)$$

Moreover, if average inflation is equal to the target and the average output gap is zero, average nominal interest rates are just the neutral real rate $\bar{r}$ plus the inflation target $\bar{\pi}$ .

$$i = \bar{r} + \bar{\pi} \qquad (8)$$

Assume now that the inflation target for period (t+1) changes from $\bar{\pi}$ to $\bar{\pi}-\varepsilon$ , a lower target. Some could fear that imposing a stricter target would lead to the need for actual output to reach below potential (for nominal and real rates to increase), but this view ignores the reaction of inflation expectations, which – as we should see now – would lead to a very different outcome.

Indeed, under the new target in period (t+1) the solution would be the same as we have summarized in equation (6) above: the expectations (in t+1) about inflation in period (t+1) would simply be the new target:

$$E_{t+1}\pi_{t+1} = \bar{\pi} - \varepsilon \qquad (6a)$$

But then, if we know that in period (t+1) inflation expectations are equal to the new target to be implemented in that period, current expectations about future expectations are also equal to the new target.

$$E_t[E_{t+1}\pi_{t+1}] = E_t\pi_{t+1} = \bar{\pi} - \varepsilon \qquad (9)$$

Replacing the expression for current expectation of inflation in (t+1) (9) into equation (5) we find a new expression for current inflation expectations:

$$E_t\pi_t^N = (1-\omega)\bar{\pi} + \omega\bar{\pi} - \omega\varepsilon = \bar{\pi} - \omega\varepsilon \qquad (10)$$

In plain words, when the government announces its intention to lower the inflation target for the next period (t+1) and the Central Bank follows a rule consistent with the Taylor Principle (which implies inflation being, on average, at the target), inflation expectations for the next period decline, which leads to a decline in expectations for current inflation as well. If the current inflation target is, say, 4.5%, and the government announces that in the next year the target will be lowered to 4%, inflation expectations for the next year would already move to 4% and to something below 4.5% for the current year. Indeed, if the weight on future inflation (ω) is 25%, then current inflation expectations would drop from 4.5% to 4.375%.

That said, with current inflation expectations reaching below the target the new expected value for the output gap is no longer zero. Replacing (10) into (4) we find that the new expected output gap is positive:

$$E_t y_t^N = \frac{\beta a\omega\varepsilon}{1+b\beta} > 0 \qquad (11)$$

Consistent with that, the new real interest rate also reaches levels below neutral.

$$i_t^N = \bar{r} + \bar{\pi} - \frac{(1+b\beta+a)}{1+b\beta}\omega\varepsilon \qquad (12)$$

That is, contrary to a common sense view, the announcement of a lower future target for inflation can actually lead to lower current interest rates and higher current output. The key to understanding this result lies, as we have attempted to make clear, in the behavior of inflation expectations. To the extent that inflation expectations in the next period drop (since the Central Bank is expected to reach, on average, the inflation target), current inflation expectations also decline, since they reflect not only the current target but also expectations about future inflation.

Yet, as current inflation expectations decline, the Central Bank – following its Taylor rule – reduces nominal interest rates. As the Central Bank reacts to changes in inflation expectations by more than 1:1,[3] the reduction of inflation expectations translates into a larger cut in nominal rates (that is, a decline of the real interest rate). As a result, actual output reaches above potential.

Of course, all these movements take place *only in the current period* (that is, they are transitory). In the next year, already under the new target, the results replicated what we have found before: expected inflation is equal to the (new) target, the expected output gap is zero, and expected nominal and real rates are at the neutral level (although now the neutral level for the nominal is lower, since the inflation target is also lower).

**The model with inflationary inertia**

The result in the previous section is quite strong. A reduction in the future inflation target can actually result in lower nominal and real interest rates and higher growth, precisely the opposite of what common sense would suggest. That said, it is important to check the robustness of the result –how it would stand under different assumptions. In this section we address the possibility of current inflation being affected by past inflation (through, for instance, indexation clauses), or, in other words, *inflationary inertia*.

Again, at first glance one could imagine that the presence of inflationary inertia would lead to a very different result, as the impact of past inflation could creep into inflation expectations and, therefore, prevent the positive effects of a lower future

[3] Note that, according to inflation (10), a decline of inflation expectations amounting to ωε becomes a decline in nominal rates equivalent to (1+a)ωε.

inflation target. Yet, as we intend to show, the positive result derived above remains, although the calculation can be somewhat more cumbersome.

In practice, we maintain both the IS relation (2) and the Taylor rule (3) from the previous model, but assume a more general form of the Phillips curve, which now explicitly allows for past inflation ($\pi_{t-1}$) and inflation expectations ($E_t\pi_{t+1}$) affecting current inflation.

$$\pi_t = \rho\pi_{t-1} + (1-\rho)E_t\pi_{t+1} + \alpha y_t + e_t \,;\; 0 \le \rho < 1 \tag{13}$$

The coefficient ρ captures the degree of inflation persistence. The higher ρ is, the more persistent inflation is, so the weight of past inflation on current inflation is higher, and the less important inflation expectations become. Conversely, the lower the persistence of inflation, the higher the weight on inflation expectations. Indeed, as ρ approaches zero, equation (13) approaches equation (1) and we would be back to the previous model, which can be, thus, considered a special case of the present version.

Since both equations (2) and (3) are the same, the expected output gap is still given by equation (4), replicated below.

$$E_t y_t = \frac{-\beta a}{1+b\beta}(E_t\pi_t - \overline{\pi}) \tag{4}$$

Recall that equation (4) states that the expected output gap should be positive (actual output above potential) when *current* inflation expectations reach below the *current* inflation target, and negative if current inflation expectations reach higher than the current target.

Taking expectations of (13) and using the expression for the expected output gap above we can express current inflation expectations as a weighted average of past inflation, expected future inflation, and the inflation target.

$$E_t\pi_t = \omega\rho\pi_{t-1} + \omega(1-\rho)E_t\pi_{t+1} + (1-\omega)\overline{\pi} \tag{14}$$

where:

$$\omega\rho + \omega(1-\rho) + (1-\omega) \equiv 1 \tag{15}$$

While equation (14) displays a certain similarity to (5), its solution is far more complicated, so we present it in the Technical Appendix. Under the presence of inflationary inertia the expectation for current inflation is no longer equal to the target, but rather a weighted average between inflation in the previous period and the inflation target.[4]

$$E_t\pi_t = \lambda_1\pi_{t-1} + (1-\lambda_1)\overline{\pi} \,;\, \lambda_1 < 1 \tag{16}$$

Thus, if inflation in period (t-1) reached above the target, inflation expectations for the current period should be above the target as well. Note, however, that, since $\lambda_1<1$, eventual discrepancies between expected inflation and the target tend to die out over time, and expectations converge to the target.

The richer dynamics in the presence of inflationary inertia could suggest that reducing the future target would not help in terms of lowering current inflation expectations, particularly if past inflation was above the target (as was the case in Brazil in 2008), since inflation expectations would be (everything else constant) above the target. This, however, is false.

Even if past inflation was above the current target, a reduction in the future inflation target still has some impact on current inflation expectations. Indeed, if the target is reduced from $\overline{\pi}$ to $\overline{\pi} - \varepsilon$ from period (t+1) until the end of time, it can be shown (in the appendix) that the new current inflation expectation would be:

$$E_t\pi_t^N = \lambda_1\pi_{t-1} + (1-\lambda_1)\overline{\pi} - \frac{(1-\lambda_1)\lambda_1(1-\rho)}{\rho}\varepsilon < E_t\pi_t \tag{17}$$

This value is unequivocally lower than the value implied by equation (16). Thus, even if current inflation expectations are above the target, by setting a lower target in the future the government can still reduce current expectations and lead to a lower path of nominal and real interest rates, producing higher output than it would have reached under a higher target.

Indeed, using equation (4) above it is not difficult to see that the new expectation of the output gap is higher than the previous expectation. This, of course, does not mean that the expectation for the output gap is positive. If past inflation has deviated significantly from the target, the output gap may be negative in both cases. Yet, as indicated by the equation below, even in this case the output gap under the lower target would be less negative.

[4] The weight $\lambda_1$ is determined in the appendix.

$$E_t y_t^N - E_t y_t = \frac{-\beta a}{1+b\beta}(E_t \pi_t^N - E_t \pi_t) > 0 \quad (18)$$

Consistent with that, both nominal and real interest rates must be lower as well. Even under the assumption that inflation may be persistent, due to indexation practices, for example, it remains true that, contrary to the fears expressed by some members of the Brazilian government, the reduction of the inflation target for 2011 can actually lead to a positive (or less negative) performance in terms of output, and to lower nominal and real interest rates.

### Conclusion

Although the complexity of the issue, in particular the need to keep track of the various impacts through different channels, requires a more technical approach, we believe the reasoning behind the results in both sections is straightforward and can be conveyed in plain words (after, of course, the model has taught us what is going on).

A reduction in the future inflation target is bound to have impacts on current inflation expectations. The key for that channel is the notion that companies set prices at some interval, and, in doing so, have to account for the evolution of future inflation[5] when deciding on the current price setting. To the extent that the Central Bank is actually able to anchor inflation expectations at the target (a feat that can be accomplished by following a monetary policy rule akin to the one we presented above), a lower (and credible) target for future inflation trickles down to lower current inflation expectations.

But the Central Bank itself makes decisions about interest rates based on current inflation expectations. Moreover, the rule that anchors inflation expectations requires the Central Bank to “overreact” to changes in inflation expectations –to change rates by more than the change in inflation expectations. Thus, as inflation expectations fall, motivated by a lower future target, nominal interest rates drop even more, leading to a decline in real interest rates and to a temporary increase in current output.

In a more general setting (that is, in the model with persistent inflation) the increase in output need not lead to a positive output gap, as noted above, if past inflation has exceeded the target. Yet, looking at the Brazilian case, both the Central Bank and the market consensus suggest that inflation in 2010 should be lower than the target, indicating that even this problem should not materialize. At any rate, however, even if this were not the case, our results suggest that a lower future target can contribute to current expansion.

Thus, contrary to popular belief and the apparent position sponsored by the Finance Minister, a lower future inflation target can give some help in terms of accelerating the economic recovery. Based on this reasoning, we believe Brazil is in an excellent position to push for more ambitious targets for inflation, which can accelerate the convergence of Brazilian inflation to levels observed in most of the developed world.

### Technical Appendix

The problem is to solve the expectational second difference equation:

$$E_t \pi_t = \omega\rho\pi_{t-1} + \omega(1-\rho)E_t \pi_{t+1} + (1-\omega)\overline{\pi} \quad \text{(A1)}$$

There are two approaches that can be used: the method of the undetermined coefficients, or the method of factorization.[6] We use the second.

Let L be the lag operator, such that:

$$Lx_t = x_{t-1} \quad \text{(A2)}$$

Conversely, we can define the forward operator, $F=L^{-1}$, such that:

$$Fx_t = x_{t+1} \quad \text{(A3)}$$

Using these definitions we can rewrite (A1) as:

$$E_t \pi_t = \omega\rho L E_t \pi_t + \omega(1-\rho)F E_t \pi_t + (1-\omega)\overline{\pi} \quad \text{(A4)}$$

Rearranging terms, this equation becomes:

$$\left[\omega(1-\rho)F - 1 + \omega\rho\right]E_t \pi_t = -(1-\omega)\overline{\pi}$$

or:

[5] Technically speaking, the key assumption that drives our results is the hypothesis that current inflation depends on future inflation.
[6] A step-by-step description of both methods can be found in the appendix to chapter 5 in Olivier Blanchard and Stanley Fischer, *Lectures on Macroeconomics*, MIT Press, 1989.

$$\left[F^2 - \frac{1}{\omega(1-\rho)}F + \frac{\rho}{1-\rho}\right]\omega(1-\rho)LE_t\pi_t = -(1-\omega)\overline{\pi} \qquad \text{(A5)}$$

The expression between brackets can be factorized as:

$$(F-\lambda_1)(F-\lambda_2) \qquad \text{(A6)}$$

where $\lambda_1$ and $\lambda_2$ are the roots[7] to the second degree equation:

$$\left[\lambda^2 - \frac{1}{\omega(1-\rho)}\lambda + \frac{\rho}{1-\rho}\right] = 0 \qquad \text{(A7)}$$

such that

$\lambda_1\lambda_2 = \rho/(1-\rho)$ and $\lambda_1 + \lambda_2 = 1/\omega(1-\rho)$ ; $\lambda_1 <1$ and $\lambda_2>1$ (A8)

Using (A6), (A5) can be re-written as:

$$[(F-\lambda_1)(F-\lambda_2)]\omega(1-\rho)LE_t\pi_t = -(1-\omega)\overline{\pi}$$

Hence:

$$(F-\lambda_1)\omega(1-\rho)LE_t\pi_t = \frac{-(1-\omega)\overline{\pi}}{(F-\lambda_2)} \qquad \text{(A9)}$$

We now proceed to work on the right hand side of (A9):

$$\frac{-(1-\omega)\overline{\pi}}{(F-\lambda_2)} = \frac{(1-\omega)\overline{\pi}}{\lambda_2(1-F\lambda_2^{-1})} = \frac{(1-\omega)}{\lambda_2}\frac{1}{(1-F\lambda_2^{-1})}\overline{\pi} = \frac{(1-\omega)}{\lambda_2}\sum_{j=0}^{\infty}\left(F\lambda_2^{-1}\right)^j\overline{\pi} \qquad \text{(A10)}$$

If the inflation target is constant, (A10) becomes:

$$\frac{-(1-\omega)\overline{\pi}}{(F-\lambda_2)} = \frac{(1-\omega)\overline{\pi}}{(\lambda_2 - 1)} \qquad \text{(A11)}$$

Using the expression (A11) into (A9) we find:

$$(F-\lambda_1)\omega(1-\rho)LE_t\pi_t = \frac{(1-\omega)\overline{\pi}}{(\lambda_2 - 1)}$$

Developing the terms in the left-hand side the expression above turns into:

$$E_t\pi_t = \lambda_1\pi_{t-1} + \frac{(1-\omega)\overline{\pi}}{\omega(1-\rho)(\lambda_2 - 1)} \qquad \text{(A12)}$$

While (A12) might look like a dead end, it can be shown that:

$$\frac{(1-\omega)\pi}{\omega(1-\rho)(\lambda_2 - 1)} = (1-\lambda_1) \qquad \text{(A13)}$$

Indeed:

$$(1-\lambda_1)(\lambda_2 - 1) = \lambda_2 - 1 - \lambda_1\lambda_2 + \lambda_1$$

Using, however, the properties displayed in (A8) and identity (15) in the text we find that:

$$(1-\lambda_1)(\lambda_2 - 1) = \frac{1-\omega\rho-\omega(1-\rho)}{\omega(1-\rho)} = \frac{1-\omega}{\omega(1-\rho)}$$

That is:

[7] It is possible to show that the roots of (A7) are real.

$(1-\lambda_1) = \frac{1-\omega}{\omega(1-\rho)(\lambda_2 - 1)}$, as claimed.

Hence (A12) now becomes:

$$E_t \pi_t = \lambda_1 \pi_{t-1} + (1-\lambda_1)\overline{\pi} \qquad \text{(A14)}$$

In plain words, the current inflation expectation is a weighted average of past inflation and the inflation target. In the particular case in which there is no persistence (ρ=0) the second degree equation (A7) becomes:

$$\lambda^2 - \frac{1}{\omega}\lambda = 0 \qquad \text{(A7a)}$$

and the solution would be:

$$\lambda_1 = 0; \lambda_2 = (1/\omega) > 1$$

Thus (A14) becomes:

$$E_t \pi_t = \overline{\pi}$$

That is, the same solution as in the case of no inflationary inertia, showing that this is indeed a particular case of the model with inflation persistence.

If the target is no longer constant, but instead $\overline{\pi}$ for the current period and $\overline{\pi} - \varepsilon$ from period (t+1) until Judgment Day, the solution to the infinite series in (A10) becomes:

$$\sum_{j=0}^{\infty} \left(F\lambda_2^{-1}\right)^j \overline{\pi} = \frac{\lambda_2 \overline{\pi}}{(\lambda_2 - 1)} - \frac{\varepsilon}{(\lambda_2 - 1)} \qquad \text{(A15)}$$

Using this expression and following the same steps as before we can express the new current inflation expectation as:

$$E_t \pi_t^N = \lambda_1 \pi_{t-1} + (1-\lambda_1)\overline{\pi} - \frac{(1-\lambda_1)\lambda_1(1-\rho)}{\rho}\varepsilon \qquad \text{(A16)}$$

This shows that the reduction in the future inflation target pushes current expectations down, the same result as in the model without inflationary inertia, although the intensity is not the same.

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## Fixed Income Research

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